Appendix: Pattern Detection Using a Variant of RANSAC Algorithm

Algorithm 1 Alignment Relation Detection

1: Obtain the edge set $\mathcal{E}$
2: **while** $\mathcal{E}$ is not empty **do**
3:     **for all** $e \in \mathcal{E}$ **do**
4:         Let the line $\mathcal{P} = e$, the inlier set $\mathcal{S} = \emptyset$
5:         **do**
6:             Let current inlier set $\mathcal{S}' = \mathcal{S}$
7:             Find the new inlier set $\mathcal{S}$ according to $\mathcal{P}$
8:             Refit the line $\mathcal{P}$ according to $\mathcal{S}$
9:         **while** $\mathcal{S}' \neq \mathcal{S}$
10:     **end for**
11:     Find the line $\mathcal{P}$ with the largest inlier set $|\mathcal{S}|$ and store it, Let $\mathcal{E} = \mathcal{E} - \mathcal{S}$
12: **end while**

We first describe how we estimate a line $\mathcal{P}_i$ from a set of inliers $\mathcal{S}_i$ at iteration i (Step 3). For a line $\mathcal{P}_i$, we need to estimate its position $\mathbf{p}(\mathcal{P}_i)$ and direction $\mathbf{d}(\mathcal{P}_i)$. $\mathbf{p}(\mathcal{P}_i)$ can be considered as the coordinate of a point on this line. First, we estimate $\mathbf{d}(\mathcal{P}_i)$ using a simple PCA based method. We consider each inlier $s \in \mathcal{S}_i$ as a weighted point whose coordinate $\mathbf{p}(s)$ is the center of edge s and the weight is $l(s)$ which is the length of the inlier edge s. $\mathbf{d}(\mathcal{P}_i)$ is obtained by computing the first principal component of these weighted points. In 2D case, if the direction of the line $\mathcal{P}_i$ is constrained, i.e., horizontal or vertical, then the direction is not modified. The position $\mathbf{p}(\mathcal{P}_i)$ is estimated as the weighted mean of the coordinates of all inliers in $\mathcal{S}_i$. Specifically, the weight of each inlier $s \in \mathcal{S}_i$ is defined as:

$$w(s) = e^{-||(\mathbf{p}(\mathcal{P}_{i-1}) - \mathbf{p}(s)) \times \mathbf{d}(\mathcal{P}_{i-1})||^2 / l(s)^2},$$

where $\mathbf{d}(\mathcal{P}_{i-1})$ and $\mathbf{p}(\mathcal{P}_{i-1})$ is the position and direction of $\mathcal{P}_{i-1}$ estimated in the previous iteration. This Gaussian weighting scheme indicates that the contribution of an inlier to the new line is higher if the inlier is longer or closer to the previous line. Then the position $\mathbf{p}(\mathcal{P}_i)$ of the new estimated line $\mathcal{P}_i$ is computed as:

$$\sum_{s \in \mathcal{S}_i} \bar{w}(s)\mathbf{p}(s),$$

where $\bar{w}(s)$ is the normalized weight.

Now we give the details of Step 2: how to get an updated set of inliers $\mathcal{S}_{i+1}$ given the line $\mathcal{P}_i$. We introduce an adaptive inlier range for robust inliers detection. Specifically, an edge t is added into $\mathcal{S}_{i+1}$ if and only if:

$$||(\mathbf{p}(t) - \mathbf{p}(\mathcal{P}_i)) \times \mathbf{d}(\mathcal{P}_i)|| \leq r,$$
$$|\mathbf{d}(t) \cdot \mathbf{d}(\mathcal{P}_i)| \geq \theta.$$

These two equations constrain that the inlier should be proximate to and have similar direction with the previous line $\mathcal{S}_i$. In the above equations, θ is set as 0.95 and $r = \min(\overline{r}(1+\sigma)h(t), 2\overline{r})$ is adaptively determined. Here $\overline{r}$ is fixed as the average size of the input elements, multiplied by a fixed factor (0.125 in our implementation). We also have:

$$\sigma = \sqrt{\sum_{s \in \mathcal{S}_i} \bar{w}(s)||(\mathbf{p}(s) - \mathbf{p}(\mathcal{P}_i)) \times \mathbf{d}(\mathcal{P}_i)||^2},$$

which measures the variance of the coordinates among the inliers in $\mathcal{S}_i$. This changes the range adaptively with respect to the degree of alignment of the previous set of inliers. The term $h(t)$ is motivated by the Gestalt law of proximity: objects that are near to one another are perceived as belonging together as a unit. We thus define:

$$h(t) = e^{-\hat{d}_t^2}$$

where $\hat{d}_t$ measures the distance orthogonal to the direction of $\mathcal{P}_i$ between t and the inliers in $\mathcal{S}_i$, normalized by the average distance between neighboring elements in the input layout. When t is away from the previous set of inliers, it is less likely to be detected as an inlier.

The following pseudo codes briefly illustrate the alignment relation detection procedure.